

## Word Problems and Equations

Prepared by Johnny Howard © 2015 South-Western, a part of Cengage Learning

## Learning Objectives

By studying this chapter and completing all assignments, you will be able to:

Learning Objective 1 Use a systematic approach to solve word problems.

Learning Objective $\mathbf{2}$ Apply formulas to solve rate, time, and distance problems.

Learning Objective 3 Solve simple numerical equations.

Learning Objective
4 Recognize numerical relationships in a series.

Learning Objective 5
Do quick mental calculations through a process of rounding numbers.

1. Read the problem carefully.
2. Determine what is requested.
3. Determine the processes to use.

## EXAMPLE A

A company orders carpeting for three offices measuring 15 square yards, 15 square yards, and 10 square yards, respectively. A carpet dealer sells the carpet for $\$ 10$ a square yard and gives a $\$ 50$ discount when the sale is for three or more offices. How much would the company pay to have the three offices carpeted?

STEP 1 Read the problem completely and carefully.
STEP 2 Determine what is requested: How much money would the company pay?
STEP 3 Determine the processes to use. Add square yards in the three offices: $15+15+10=40$.
Multiply the total square yards by the $\$ 10$ per square yard cost: $40 \times \$ 10=\$ 400$.
Subtract the $\$ 50$ discount: $\$ 400-\$ 50=\$ 350$.

## EXAMPLE B

Phoebe Elias owns half of a small bakery. Last week she baked 6 cakes on Monday, 9 on Tuesday, 11 on Wednesday, 8 on Thursday, and 6 on Friday. She sold all cakes for $\$ 9$ each. It cost Phoebe $\$ 5$ to make each cake; the rest was her profit on each cake.

Phoebe split her profit evenly with her partner. How much did her partner receive from last week's cakes?

STEP 1 Read the problem completely and carefully.
STEP 2 Determine what is requested: How much money did Phoebe's partner receive?
STEP3 Determine the processes to use.
Add the cakes baked: $6+9+11+8+6=40$.
Subtract the cost from the sales price: $\$ 9-\$ 5=\$ 4$ profit per cake.
Multiply the number of cakes sold by the $\$ 4$ profit per cake: $40 \times \$ 4=\$ 160$.
Divide the total profit by 2 : $\$ 160 \div 2=\$ 80$ received by the partner.

STEPS to Solve Rate, Time, and Distance Problems

1. Rate, time, and distance problems are solved with a simple formula: Rate (speed) $\times$ Time $=$ Distance (amount done).
2. If you are given any two factors, it is easy, by formula, to find the third:

Rate $\times$ Time $=$ Distance
Distance $\div$ Time $=$ Rate
Distance $\div$ Rate $=$ Time

## EXAMPLES C, D, E and F

Jan traveled at 35 miles per hour for 5 hours. How far did Jan travel?

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35 mph }\times5\textrm{hr}=175\textrm{mi
(Rate \(\times\) Time \(=\) Distance)
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Jan traveled 175 miles in 5 hours. How fast was Jan traveling?
$175 \mathrm{mi} \div 5 \mathrm{hr}=35 \mathrm{mph}$
(Distance $\div$ Time $=$ Rate)
At 35 miles per hour, how long would it take Jan to travel a total of 175 miles?
$175 \mathrm{mi} \div 35 \mathrm{mph}=5 \mathrm{hr}$
(Distance $\div$ Rate $=$ Time)
Jan and Ahmed start traveling toward each other from 300 miles apart. Jan is traveling at 35 miles per hour; Ahmed is traveling at 40 miles per hour. How much time will elapse before they meet?
Distance $=300 \mathrm{mi}$
Total rate $=35 \mathrm{mph}(\mathrm{Jan})+40 \mathrm{mph}(\mathrm{Ahmed})=75 \mathrm{mph}$
$300 \mathrm{mi} \div 75 \mathrm{mph}=4 \mathrm{hr}$
(Distance $\div$ Rate $=$ Time)

1. A numerical sentence in which both sides of an equal sign contain calculations is called an equation.
2. For an equation to be true, the numbers on the left of the equal sign must always compute to the same answer as the numbers on the right of the equal sign.
3. Moving a number from one side of the equation to the other changes its sign:

- A plus sign will change to minus.
- A minus sign will change to plus.
- A multiplication sign will change to division.
- A division sign will change to multiplication.


## EXAMPLEK

$6+4+5=17-2$
Change the -2 :
$6+4+5+2=17$

Now change the +5 : (Move the positive 5 to the other side of the equation.)

## Addition-Subtraction

(Move the negative 2 to the other side of the equation.)
$6+4+2=17-5$

## EXAMPLE L <br> Multiplication-Division

Change the $\div 2$ :
$3 \times 8 \times 2=48$
Now change the $\times 8$ :
$3 \times 2=48 \div 8$
$3 \times 8=48 \div 2$

## EXAMPLE M

$6+2=5+?$
Change a number
$6+2-5=$ ?
Therefore, $?=3$

## EXAMPLE O

$7+3+6=4+4+$ ?
Change a number
$7+3+6-4-4=$ ?
Therefore, $?=8$

## EXAMPLE N

$$
15-3=2+?
$$

Change a number $15-3-2=$ ?
Therefore, ? $=10$

## EXAMPLE P

$20 \div 5=2 \times$ ?
Change a number $20 \div 5 \div 2=$ ?
Therefore, ? $=2$

## EXAMPLEQ

Last year a company had sales of $\$ 25,000$ in Dept. A and $\$ 20,000$ in Dept. B. If sales this year were $\$ 30,000$ in Dept. A, what is the amount needed for Dept. B to equal last year's sales?

Last year: Dept. A $\$ 25,000+$ Dept. B $\$ 20,000=\$ 45,000$
This year: Dept. A $\$ 30,000+$ Dept. B ? $\quad=\$ 45,000$
Dept. B $=\$ 45,000-$ Dept. A $\$ 30,000$
Therefore, ? $=\$ 15,000$

1. Relationships in a series of numbers may be found by comparing the first three or four terms in a series and then extrapolating the numbers that would most logically come next.
2. Recognizing numerical and series relationships can be important in analyzing, communicating, and computing numbers. These relationship series are also used frequently in initial employment tests.

STEPS to Making Quick Calculations by Rounding Numbers

1. Quick calculations are beneficial when working in business situations.
2. Rounding odd and difficult-to-compute amounts to even whole numbers that are easier to compute is a technique often used in business.
3. By rounding, you will be able to get quick and accurate answers without having to write out the computations.

## EXAMPLES R, S and T

How much would 5 items at $\$ 2.99$ each cost?
To make this computation easily, think " $\$ 2.99$ is $\$ 0.01$ less than $\$ 3.00$." Then think " 5 times $\$ 3$ equals $\$ 15$." Finally, think " $\$ 15.00$ less $\$ 0.05(5 \times \$ 0.01)$ is $\$ 14.95$," which is the correct answer.

The total cost of three equally priced dresses is $\$ 119.85$. How much does each dress cost?
To figure out this problem easily, think " $\$ 119.85$ is $\$ 0.15$ less than $\$ 120.00$." Then think " $\$ 120$ divided by $3=\$ 40$, and $\$ 40.00$ less $\$ 0.05(\$ 0.15 \div 3)$ is $\$ 39.95$," the correct answer.

At 19 miles per gallon, how many miles would a car go on 9 gallons of gas?
To figure out this problem easily, think " 19 is just 1 mile less than 20 ." Then think
" 9 times $20=180$, and 180 minus $9(9 \times 1)$ is 171 ," the correct answer.

## Chapter Terms for Review

## equation

numerical sentence

## Assignment 3.1: Word Problems, Equations, and Series

## (A) Use the three-step process to solve the following word problems.

1. Budget Lamps and Lighting, Inc., conducted a direct-mail program. The manager determined that $\$ 72,000$ in new business came from the program. If the profit was $40 \%$ of sales, how much profit did the program produce? $\$ 28,800$
$\$ 72,000 \times 0.40=\$ 28,800$
2. Martha's Beauty Salon charges $\$ 40$ for a haircut, $\$ 48$ for a facial, and $\$ 60$ for hair coloring. If it had 20 haircut, 22 facial, and 18 coloring customers, what were its total sales? $\$ 2,936$
$(20 \times \$ 40)+(22 \times \$ 48)+(18 \times \$ 60)=\$ 2,936$
3. Juan Lopez sold 11 life insurance policies with premiums totaling $\$ 24,200$. He sold 15 auto policies with premiums totaling $\$ 34,200$. Which type of policy had the greater premium per sale? AUTO LIFE: $\$ 24,200 \div 11=\$ 2,200 \quad$ AUTO: $\$ 34,200 \div 15=\$ 2,280$
4. The Tulsa Taxi Service had four taxi vehicles. Two got 25 miles per gallon of gas; two got 20 miles per gallon of gas. The vehicles were each driven 8,000 miles per month. The gas cost $\$ 3.80$ per gallon. What was the amount of the gas bill for the month? $\$ 5,472$
$8,000 \div 25 \times 2 \times \$ 3.80=\$ 2,432 \quad 8,000 \div 20 \times 2 \times \$ 3.80=\$ 3,040 \quad \$ 2,432+\$ 3,040=\$ 5,472$

## Assignment 3.1: Word Problems, Equations, and Series

B Do these problems without using scratch paper or an electronic calculator.
5. How much would you pay for 12 gallons of gasoline selling at $\$ 4.29$ per gallon? $\$ 51.48$
6. How many items would you have if you had 98 books, 98 cards, and 98 pencils? 294
7. What is the price of 15 items at $\$ 2.99$ each? $\$ 44.85$
8. How much would you have if you received $\$ 3.99$ from one person, $\$ 7.99$ from a second, $\$ 11.99$ from a third, and $\$ 1.99$ from a fourth? $\$ 25.96$
9. If 24 people were divided into three equal groups and each group added 2 additional members, how many members would be in each group? 10

## Assignment 3.1: Word Problems, Equations, and Series

C Do the steps in the order in which they occur. Do these problems without using scratch paper or an electronic calculator.
10. 12 items at $\$ 3$ each plus $\$ 2$ tax $=\$ 38$
11. 15 watches at $\$ 30$ each less a $\$ 50$ discount $=\$ 400$
12. 3 lamps at $\$ 22$ each plus 7 bulbs at $\$ 2$ each $=\$ 80$
13. 100 belts at $\$ 10$ each less discounts of $\$ 60$ and $\$ 30=\$ 910$
14. 3 dozen scissors at $\$ 11.20$ per dozen plus a $\$ 4$ shipping charge $=\$ 37.60$
15. 6 pounds of pears at $\$ 3$ per pound plus $50 \Phi$ per pound for packaging $=\$ 21$
16. $\$ 38$ sale price plus $\$ 3$ tax less an $\$ 11$ discount plus a $\$ 5$ delivery charge $=\$ 35$
17. 6 bath towels at $\$ 8$ each and 4 hand towels at $\$ 3$ each plus $\$ 2.50$ tax $=\$ 62.50$
18. 4 dozen brushes at $\$ 25$ per dozen plus $\$ 5$ tax plus $\$ 7$ shipping charge $=\$ 112$
19. 2 shirts at $\$ 30$ each, 4 ties at $\$ 10$ each, and 7 pairs of socks at $\$ 2$ each $=\$ 114$

## Assignment 3.1: Word Problems, Equations, and Series

D Complete the following equations by supplying the missing items.
20. $42+8=\underline{41}$ $+4=4+16$
24. $9+17-3=4 \times \underline{7}-5$
26. $13-11 \times \underline{40}=8 \times 8+16$
28. $68 \div 2=544 \div 16$
21. $13+22=2+33$
23. $400=17-2+385$
25. $160 \div 4+2=7 \times 7-7$
27. $3 \times 3 \times 3=9 \div 3 \times 9$
29. $32-12+7=30-3$

## Assignment 3.1: Word Problems, Equations, and Series

E In each of the following problems, a definite relationship exists among the numbers in each series. Extend each series two items by following the correct process.
30. Extend each series below through addition.
a. $4,8,12,16,20,24$
b. $1,4,5,8,9,12$
c. $5,8,10,13,15,18,20$
31. Extend each series below through subtraction.
a. $50,45,40,35,30,25$
b. $60,55,53,48,46,41$
c. $100,90,81,73,66,60$
32. Extend each series below through multiplication.
a. $4,8,16,32,64,128$
c. $2,4,20,40,200,400$
b. $5,25,125,625,3,125$ $\qquad$
33. Extend each series below through division.
a. $15,625,3,125,625,125,25,5$
c. $10,000,2,000,1,000,200,100,20$
b. $729,243,81,27,9,3$
34. Extend each series below through combinations of the four processes above.
a. $72,75,69,72,66,69$
b. $200,100,300,150,450,225$
c. $5,7,14,16,32,34,68$
d. $240,120,600,300,1,500,750,3,750$
e. $7,4,8,5,9,6,10$
f. $30,10,60,20,120,40$
g. $10,40,20,80,40,160$
h. $100,50,40,20,10,5$

## Assignment 3.2: Word Problems, Formulas, and Equations

## A Solve the following word problems.

1. A store regularly sold 2 cans of soup for $\$ 1.30$. It advertised a special sale of 6 cans for $\$ 3.12$. A customer bought 12 cans at the sale. How much did the customer save over the regular price? $\$ 1.56$
$\$ 1.30 \div 2=\$ 0.65$ each
$\$ 0.65-\$ 0.52=\$ 0.13$
$\$ 3.12 \div 6=\$ 0.52$ each
$\$ 0.13 \times 12=\$ 1.56$ saved
2. A sales representative's car gets 18 miles to a gallon of gas. It was driven 120 miles each day for 30 days. Gas cost an average of $\$ 3.68$ per gallon. What was the sales representative's total 30 -day cost for gas? $\$ 736$
$120 \times 30=3,600 \mathrm{mi}$
$3,600 \div 18=200 \mathrm{gal}$
$200 \times \$ 3.68=\$ 736$
3. A store clerk sold a customer a ruler for $\$ 1.67$, three pencils for $\$ 0.23$ each, notebook paper for $\$ 0.99$, and an eraser for $\$ 0.35$ and was given $\$ 10.00$ in payment. How much change did the clerk give the customer from the $\$ 10.00$ ? (All prices include tax.) $\$ 6.30$
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$1.67 + (3 > $0.23) + $0.99 + $0.35 = $3.70
$10.00-$3.70 = $6.30
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4. A college student worked at a local store for $\$ 9.00$ per hour, as his class schedule permitted. The student worked 3 hours each Monday, Tuesday, Wednesday, and Thursday. He also worked 2 hours each Friday and 8 hours each Saturday. How many weeks did the student have to work to earn $\$ 792$ for a new bicycle?
4 weeks
$(3 \times 4)+2+8=22$ hrs per week
$\$ 9.00 \times 22=\$ 198$ per week
$\$ 792 \div \$ 198=4$ weeks

## Assignment 3.2: Word Problems, Formulas, and Equations

## A Solve the following word problems. (cont'd)

5. A box, a crate, and a trunk weigh a total of 370 pounds. The crate weighs 160 pounds. The trunk weighs 4 pounds more than the box. What does the box weigh? 103 lb
$370-160-4 \div 2=103 \mathrm{lb}$
6. A hotel has 15 floors. Each floor has 26 single-person rooms and 38 two-person rooms. What is the total guest capacity of the hotel? 1,530
$26+(38 \times 2) \times 15=1,530$ guest capacity
7. A department store offers its customers socks for $\$ 1.50$ per pair or $\$ 15.00$ per dozen. If two customers buy 1 dozen together and each pays half the cost, how much will each customer save by paying the quantity price? $\$ 1.50$
$\$ 1.50 \times 12=\$ 18.00$
$\$ 18.00-\$ 15.00=\$ 3.00$
$\$ 3.00 \div 2=\$ 1.50$ each saved
8. Supply Clerk A ordered five staplers at $\$ 9$ each and two large boxes of staples for $\$ 3$ each. Supply Clerk B ordered a box of computer disks for $\$ 8.50$ and a box of computer paper for $\$ 39.95$. How much less did Clerk B spend than Clerk A? (All prices include tax.) \$2.55
A: $(5 \times \$ 9)+(2 \times \$ 3)=\$ 51$
B: $\$ 8.50+\$ 39.95=\$ 48.45$
C: $\$ 51-\$ 48.45=\$ 2.55$

## Assignment 3.2: Word Problems, Formulas, and Equations

B Solve the following time, rate, distance problems.
9. Linda leaves Boise to travel the 2,070 miles to Austin, driving at a speed of 55 miles per hour. Mark leaves Austin to travel the same 2,070-mile route to Boise, driving at a speed of 60 miles per hour. How many miles will Mark have traveled when they meet? $1,080 \mathrm{mi}$
$2,070 \div 115(55 \mathrm{mph}+60 \mathrm{mph})=18 \mathrm{hr}$
$18 \mathrm{hr} \times 60=1,080$
10. Car A traveled to a destination 845 miles away at 65 miles per hour. Car B traveled to a destination 495 miles away at 55 miles per hour. How much longer did Car A travel than Car B? 5 hr
A: $845 \div 65=13 \mathrm{hr}$
B: $495 \div 55=9 \mathrm{hr}$
$14 \mathrm{hr}-9 \mathrm{hr}=5 \mathrm{hr}$

## Assignment 3.2: Word Problems, Formulas, and Equations

C Solve each of the problems without writing any computations on paper and without using a calculator or a computer.
11. 12 items at $\$ 1.99=\$ 23.88$
13. 4 items at $\$ 19.98=\$ 79.92$
15. 15 items at $\$ 0.99=\$ 14.85$
17. 6 items at $\$ 2.95=\$ 17.70$
19. 19 items at $\$ 40=\$ 760$
21. 20 items at $\$ 40.05=\$ 801$
23. 20 items at $\$ 39.98=\$ 799.60$
25. 48 items at $\$ 5=\$ 240$
27. 7 items at $\$ 7.97=\$ 55.79$
29. 30 items at $\$ 2.98=\$ 89.40$
12. 2 items at $\$ 5.95=\$ 11.90$
14. 2 items at $\$ 49.96=\$ 99.92$
16. 10 items at $\$ 9.99=\$ 99.90$
18. 5 items at $\$ 1.02=\$ 5.10$
20. 3 items at $\$ 19.99=\$ 59.97$
22. 30 items at $\$ 1.99=\$ 59.70$
24. 2 items at $\$ 5.99=\$ 11.98$
26. 5 items at $\$ 1.97=\$ 9.85$
28. 2 items at $\$ 99.98=\$ 199.96$
30. 99 items at $\$ 1.90=\$ 188.10$

## Assignment 3.2: Word Problems, Formulas, and Equations

D In each of the following equations, rewrite the equation by moving the last number on each side of the equal sign to the other side and making appropriate sign changes so that the equation is still true. (Example: Given $13+7+2=10+12$; Answer 13 + 7-12 = 10-2)
31. $6+4+5=17-2$
$6+4+2=17-5$
33. $9-3-3=2+1$
$9-3-1=2+3$
35. $20+1-7=16-2$
$20+1+2=16+7$
37. $12+3-5=7+3$
$12+3-3=7+5$
39. $64-32-16=8+8$
$64-32-8=8+16$
32. $6 \times 2 \div 3=8 \div 4 \times 2$
$6 \times 2 \div 2=8 \div 4 \times 3$
34. $8 \div 2 \times 4=24 \div 3 \times 2$
$8 \div 2 \div 2=24 \div 3 \div 4$
36. $3 \times 3 \times 3=18 \div 2 \times 3$
$3 \times 3 \div 3=18 \div 2 \div 3$
38. $7 \times 4 \div 2=28 \times 2 \div 4$
$7 \times 4 \times 4=28 \times 2 \times 2$
40. $63 \div 7 \times 2=3 \times 2 \times 3$
$63 \div 7 \div 3=3 \times 2 \div 2$

